From *Audio Xpress*, August 2008, pp.19-20.

Choosing Cathode Bypass Capacitors

By Merlin Blencowe and David Ivan James

With little practical information in print, it was necessary to derive this simple solution to the problem of triode cathode bypassing.

When designing a simple cathode-biased triode, voltage-gain stage of the type shown in **fig. 1**, the designer is usually faced with choosing a cathode bypass capacitor in order to maximise gain. If the cathode resistor is left unbypassed, negative current feedback will limit the available gain of the stage to some minimum value. Usually this is a simple matter of using an arbitrarily large value of capacitance to ensure proper cathode decoupling well below audible frequencies.

Occasionally though, some of you may wish to use a small value of capacitance in order to obtain a treble-boost action, because the cathode will be fully bypassed at high frequencies and effectively unbypassed at low frequencies. This might be called "partial bypassing", and results in the stage operating as a first-order shelving filter. Possible applications include phono stages or preamplifiers for musical instruments and microphones, where a bass cut is often desirable.

Calculating the maximum possible (fully bypassed) gain is easy, and is given by[1]:

$$A_{max} = \frac{\mu Ra}{Ra + ra} \qquad (1)$$

Likewise, the minimum possible (unbypassed) gain is just as easily found using[2]:

$$A_{min} = \frac{\mu Ra}{Ra + ra + Rk(\mu + 1)} \qquad (2)$$

Where, in both cases:
μ = the amplification factor of the valve.
Ra = the anode resistor.
ra = the internal anode resistance (plate resistance).
Rk = the cathode bias resistor.
(If the stage is heavily loaded by a following stage, you should substitute Ra for Ra||Rl, where Rl is the following loading resistance, which is usually a grid-leak resistor.)

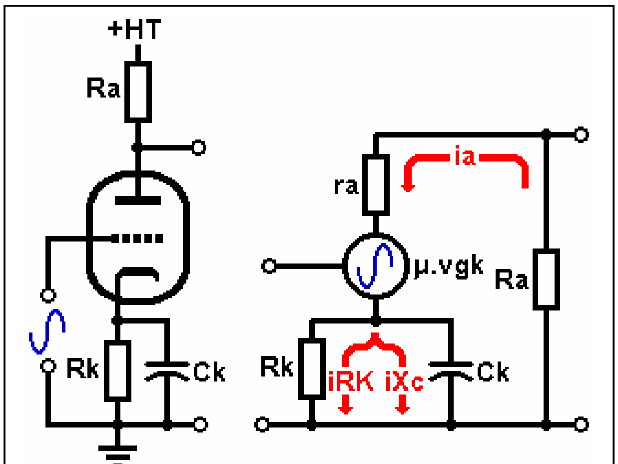

Fig. 1: Left, a simple triode amplifier stage and its Thévenin equivalent circuit, right.

LOOKING FOR ANSWERS

But at what point is the transition made from one extreme to the other? And how should you choose the cathode bypass capacitor to give the desired result?

We consulted numerous sources, but none gave satisfactory answers to these questions. The older texts all assume that an audible bass cut would be undesirable and give this subject almost no treatment at all, while some contemporary texts give incorrect treatment[3].

The *Radio Designer's Handbook* (4th ed.) does give a formula for gain at any frequency, on page 484:

$$\frac{A'}{A_{max}} = \sqrt{\frac{1 + (\omega RkCk)^2}{\left[1 + \frac{Rk(\mu+1)}{Ra + ra}\right]^2 + (\omega RkCk)}}$$

Where all symbols as above and:
A' = the gain at the frequency of interest.
$\omega = 2\pi f$.

But this is hardly a convenient formula, and even if you solved it for Ck, you must pick an arbitrary level of gain "from the air" and work from there. Of course, SPICE simulation could solve this problem without trouble, but that could be just as time consuming as solving the above formula. Therefore, we thought it would be more useful to have a universal and simple equation that would allow quick "back of an envelope" choice of the cathode bypass capacitor, or quick analysis of an existing circuit.

A SIMPLE SOLUTION

Normally, filters are defined according t the ±3dB pole/zero frequency. However, in this convention is less useful for this type of shelving filter because the difference between A_{max} and A_{min} may be much less than 6dB, in which case there will be no conventional pole/zero at all! Instead, the best way to define such a filter, which is applicable to all circumstances, is by the point at which gain is halfway between minimum and maximum, which hereafter we call the "half boost point"[4].

The problem of defining this point is most easily solved using a Nyquist plot of the denominator of equation 2 (**fig. 2**), first by drawing a vector equal to the instantaneous current through Ra and ra, and in series with this, current through Rk(μ+1). At zero frequency, no current flows in Ck, and anode current (ia) is equal to and in phase with current through Rk (iRk). At maximum gain, Ck is effectively a short circuit, and current through it (iXc) is also in phase with ia. At intermediate gain levels, is is the vector sum of iRk and iXc, the vectors of which describe a semicircular arc, as shown.

At the half boost point, you can determine the total phase shift (θ) in ia by drawing an arc of radius Ra+ra+½Rk(μ+1) as shown. Where this vector is tangential with the arc ½Rk(μ+1), then AB and BC indicate relative magnitudes of iRk and iXc respectively,

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while a/b yields the product of RkCk in radians per second, so that:
a/b = ωRkCk
It is then a simple matter of finding a/b by treating the problem as two geometrical problems as in **fig. 3**, and solving them simultaneously:
To aid algebraic manipulation, put Ra+ra=A, and ½Rk(μ+1)=B.
From elementary geometry:

$$a^2 = b(2B - b) \qquad \text{(I)}$$

and,

$$a^2 = (B - b)(2A + B + b) \qquad \text{(II)}$$

Equating I and II,

$$2Bb = 2AB + B^2 - 2Ab$$

and collecting terms in b,

$$b = \frac{B(2A+B)}{2(A+B)} \qquad \text{(III)}$$

so,

$$b^2 = \frac{B^2(2A+B)^2}{4(A+B)^2} \qquad \text{(IV)}$$

Substituting III into I and simplifying,

$$a^2 = \frac{4A^2B^2 6AB^3 + 2AB^3 + 3B^4}{4(A+B)^2}$$

$$= \frac{4B^2(A^2+2AB)+3B^4}{4(AB)^2} \qquad \text{(V)}$$

Putting V over VI and simplifying yields:

$$\frac{a^2}{b^2} = \frac{4A^2+8AB+3B^2}{(2A+B)^2} = \frac{(2A+B)+2B}{(2A+B)}$$

$$= 1 + \frac{2B}{(2A+B)}$$

so,

$$\frac{a}{b} = \sqrt{1 + \frac{2B}{(2A+B)}} \qquad \text{(VI)}$$

the limits of which are 1 and √3.

Finally, putting A=Ra+ra, and B=½Rk(μ+1) and a/b=ωRkCk,

$$\omega RkCk = \sqrt{1 + \frac{Rk(\mu+1)}{2(Ra+ra)+\frac{1}{2}Rk(\mu+1)}} \qquad \text{(VII)}$$

and solving for $f_{(half\ boost)}$:

$$f_{(halfboost)} = \frac{1}{2\pi RkCk} \cdot \sqrt{1 + \frac{Rk(\mu+1)}{2(Ra+ra)+\frac{1}{2}Rk(\mu+1)}}$$

(VIII)
Where all resistances are in Ohms and all capacitances in Farads.

Fig. 4 shows the idealised frequency response of the circuit given in fig. 1 using an ECC83 / 12AX7, constructed by drawing a first-order (6dB/octave) slope through the half boost point, found with the above equation and converting to decibels. The heavy line shows the response produced by SPICE simulation, showing excellent correlation. In fact, the accuracy of equations 1, 2 and VIII is limited only by the accuracy of the value used for ra, but this will usually be swamped by the tolerance of the capacitor and triode used. We hope you find this a useful and practical formula to accompany equations 1 and 2 given at the very beginning of this article, when designing triode amplifiers.

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[1] *Admiralty Handbook of Wireless Telegraphy*, 1938. Wireless Telegraphy Theory, vol. 2, section F4. H.M. Stationary Office, London.
[2] Langford-Smith, F.(ed.), 1953. *Radio Designer's Handbook* (4th ed.), p485. Wireless Press, Sydney.
[3] Jones, M. 2003. *Valve Amplifiers* (2nd ed.), p79. Newnes, Oxford.
[4] Langford-Smith, F.(ed.), 1953. *Radio Designer's Handbook* (4th ed.), p642. Wireless Press, Sydney.

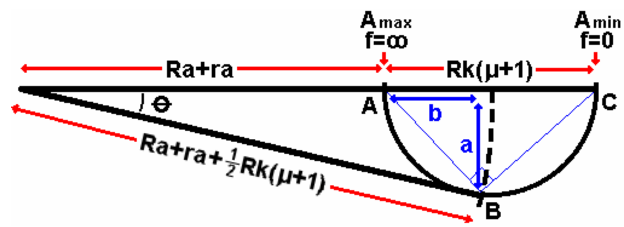

Fig. 2: A Nyquist plot of the denominator of equation 2. a/b yields the product of RkCk in radians per second.

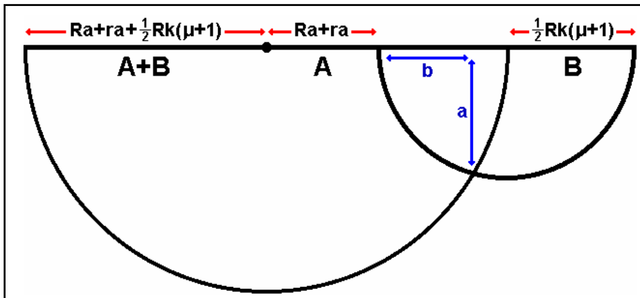

Fig. 3: Substituting the problem in fig. 2 for a simple geometrical problem.

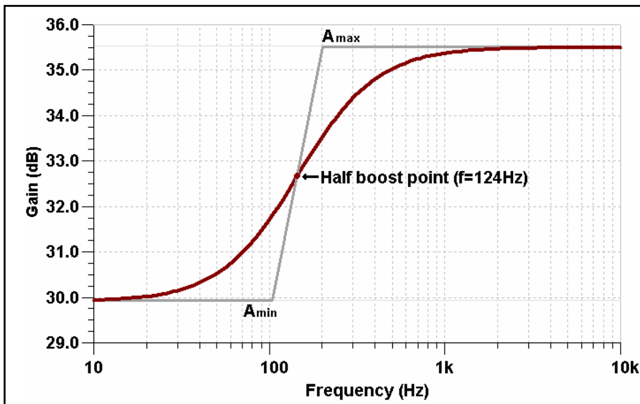

Fig. 4: Frequency response of a SPICE simulation of an ECC83 / 12AX7 in the circuit given in fig. 1, when Ra=100kΩ, Rk=1.5kΩ, Ck=1μF. A_{max}, A_{min} and the half boost point were calculated by hand using equations 1, 2 and VIII, with ra=66kΩ.