# PUSH-PULL CIRCUIT ANALYSIS*

## Cathode-Coupled Output Stage

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MOST circuits used for providing the out-of-phase signals necessary for driving push-pull stages of amplification employ a valve or centre-tapped transformer to perform the " phase-splitting " but the circuit given in Fig. 1 requires neither of these components. This circuit is one in which one output valve derives its input by cathode-coupling from the other. In its application to resistance-coupled intermediate A.F. stages it is quite well-known[1,2] but its suitability for stages having a transformer output coupling is less widely recognised. It has, however, been used in this way by C. G. Mayo and H. D. Ellis in the design of certain B.B.C. audio-frequency amplifiers.

The input signal is applied to the terminals $AB$, of which $B$ is earthed, and the two valves $V_1$ and $V_2$ (usually high-slope tetrodes or pentodes, but drawn as triodes for convenience) operate automatically in push-pull. The phase-reversal is accomplished as follows. Consider the circuit with $V_2$ and all its grid circuit components removed. The valve $V_1$ then operates normally but with some current feedback due to the A.F. potentials developed across the unbypassed grid bias resistance $R_k$ and the resistance $R_0$. These A.F. potentials provide the input signal for $V_2$ when it is in circuit.

In order that the A.C. component of the anode current of $V_2$ shall be 180 deg. out of phase with that of $V_1$, the cathode of $V_2$ must be joined to the cathode of $V_1$ and its grid must be joined to earth. This latter connection is actually made via a capacitor in order that the correct value of grid bias may be applied to $V_2$ by means of the grid leak $R_2$. Thus the anode currents of both valves pass through $R_k$ (which hence acts as a common grid bias resistance) and $R_0$ and the amplitudes of the A.C. components of these anode currents adjust themselves so that they are unequal, $V_1$ providing the greater A.C. component. This difference in amplitude is necessary in order to provide the input signal for $V_2$. It is the purpose of this analysis to find the effect of varying $R_0$ on the performance of the circuit and hence to find an expression for its optimum value.

Let $i_1$ and $i_2$ = amplitude of the A.C. components of the anode currents of $V_1$ and $V_2$ respectively (both currents are regarded as positive, their directions being indicated in Fig. 1, and $i_1$ is assumed greater than $i_2$).

$E_1$ and $E_2$ = amplitudes of the alternating potentials between grid and cathode of $V_1$ and $V_2$ respectively

$E_k$ = amplitude of the P.D. developed across $R_k$

$E_0$ = amplitude of the P.D. developed across $R_0$

$E$ = amplitude of the input signal applied across $AB$.

From Fig. 1, we have

$$\begin{aligned} E_1 &= E - (E_0 + E_k) = \\ &\quad E - [(i_1 - i_2)R_0 + (i_1 - i_2)R_k] \\ &= E - (i_1 - i_2)(R_0 + R_k) \\ &= E - (i_1 - i_2)R_0' \qquad \ldots \quad (1) \end{aligned}$$

in which $R_0' = R_0 + R_k$

$$\begin{aligned} \text{Also } E_2 &= E_0 + E_k \\ &= (i_1 - i_2)(R_0 + R_k) \\ &= (i_1 - i_2)R_0' \qquad \ldots \quad (2) \end{aligned}$$

Regarding $V_1$ and $V_2$ as constant current generators, we have

$$i_1 = g_m E_1 = g_m [E - (i_1 - i_2) R_0'] \quad (3)$$

and

$$i_2 = g_m E_2 = g_m (i_1 - i_2) R_0' \quad \ldots \quad (4)$$

in which $V_1$ and $V_2$ are assumed to be valves with the same value of $g_m$. It is considered unnecessary in equations (3) and (4) to include negative signs, thus, $i_1 = - g_m E_1$. With the input signals and anode currents of $V_1$ and $V_2$ as indicated in Fig. 1, all phase relationships are taken care of by using positive signs throughout as in equations (3) and (4).

From (3) $\quad (i_1 - i_2)R_0' = E - \dfrac{i_1}{g_m}$

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From (4) $(i_1 - i_2)R_0' = \dfrac{i_2}{g_m}$

$$\therefore \quad E - \frac{i_1}{g_m} = \frac{i_2}{g_m}$$

$$i_1 + i_2 = g_m E$$

which shows that the combined output currents of $V_1$ and $V_2$ have the same value that would be given by a single valve with the same value of $g_m$ and the same total input $E$.

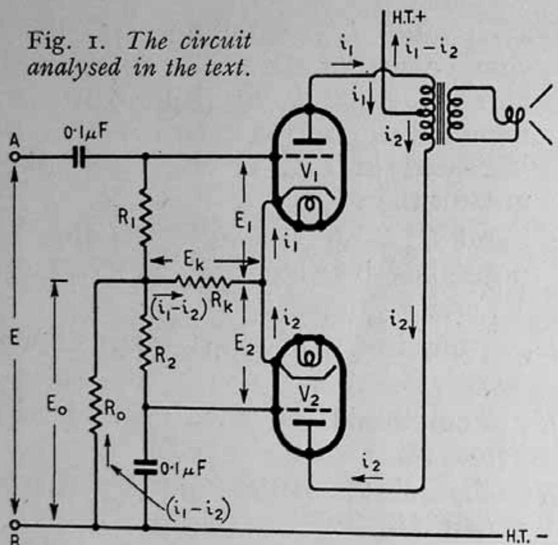


Fig. 1. *The circuit analysed in the text.*

Solving (3) and (4) for $i_1$ and $i_2$ we find

$$i_1 = \frac{(1 + g_m R_0')g_m E}{1 + 2g_m R_0'} \quad \ldots \quad \ldots \quad (5)$$

and $$i_2 = \frac{g_m{}^2 R_0' E}{1 + 2g_m R_0'} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

$$\therefore \quad i_1 - i_2 = \frac{g_m E}{1 + 2g_m R_0'} \quad \ldots \quad \ldots \quad (7)$$

In order to assess the difference between $i_1$ and $i_2$ we shall use the symbol $\Delta$ to represent (difference in amplitude of the A.C. components of the anode currents of $V_1$ and $V_2$)/(mean amplitude of the A.C. components)

i.e. $$\Delta = \frac{i_1 - i_2}{\frac{1}{2}(i_1 + i_2)}$$

From the expressions given above for $i_1 - i_2$ and $i_1 + i_2$

$$\Delta = \frac{2g_m E}{1 + 2g_m R_0'} \cdot \frac{1}{g_m E} = \frac{2}{1 + 2g_m R_0'}$$

In most practical problems it will be found that $2g_m R_0'$ considerably exceeds unity, so that

$$\Delta \approx \frac{1}{g_m R_0'}$$

$$\therefore \quad R_0' \approx \frac{1}{\Delta g_m} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Suppose, for a particular type of valve, $g_m = 10$ mA/volt and we do not wish the difference in A.C. components of the anode currents to exceed 5 per cent. of their mean value. We have, from (8).

$$R_0' \approx \frac{1}{\frac{1}{20} \cdot \frac{10}{1{,}000}} = 2{,}000 \text{ ohms.}$$

$R_0'$ is, of course, the value of total cathode resistance, i.e., grid bias resistance $R_k$, together with the coupling resistance $R_0$. If $R_k$ is 100 ohms then $R_0$ will need to be 1,900 ohms. We shall deal throughout this manuscript, however, with the value of $R_0'$ rather than $R_0$. A value of $R_0'$ of 2,000 ohms may be considered excessive, since the D.C. components of the anode currents of $V_1$ and $V_2$ which pass through it may cause the safe heater-cathode insulation rating of the valves to be exceeded. If the total cathode current of each valve is 40 mA and if $R_0'$ is 2,000 ohms, then the steady P.D. across $R_0'$ is $2{,}000 \times \frac{80}{1{,}000}$ $= 160$ volts. $R_0'$ will, therefore, need to be capable of dissipating steadily a power of $\frac{80}{1{,}000} \times 160 = 12.8$ watts, and if the valves require an anode potential of 250 volts,

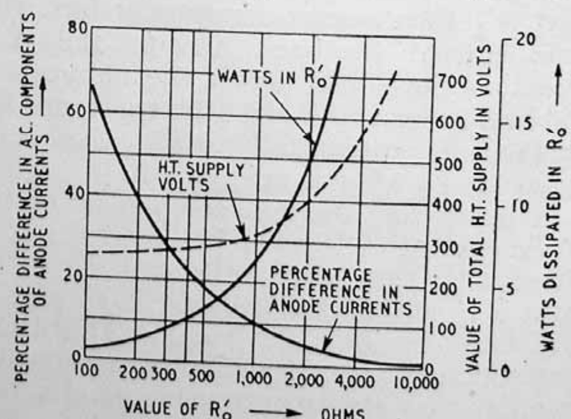


Fig. 2. *Curves illustrating the variation of H.T. supply volts, the percentage difference in anode currents and the power in $R_0'$ with variation of $R_0'$. These results apply to valves with $g_m$ = 10 mA/volt and total cathode current = 40 mA per valve.*

the total H.T. voltage necessary will be $250 + 160 = 410$, which may be considered excessive. The dependence of $\Delta$, power in $R_0'$ and H.T. voltage on the value of $R_0'$ are illustrated in Fig. 2, evaluated for valves with $g_m = 10$ mA/volt and total cathode current equal to 40 mA per valve. From this a most suitable value for $R_0'$ would appear to be about 1,000 ohms. It would be good enough, probably, to make $R_0 = 1{,}000$ ohms (a standard value), $R_k$ being 100 ohms (a typical value) so that $R_0' = 1{,}100$ ohms. Clearly, for a given value of $\Delta$, in order that the power wasted

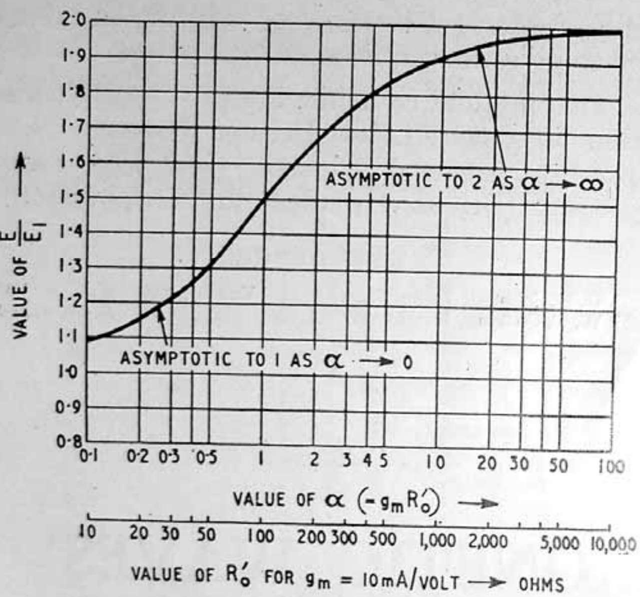


Fig. 3. *Dependence of* $E/E_1$ *on the value of* $g_mR_0'$.

in $R_0'$ shall be a minimum and also for minimum value of total H.T. supply, the highest possible value of $g_m$ is necessary. Also, since the input signal for the second valve is given by the product of $R_0'$ and $\Delta$ (see equation (2)) it is a good thing to use valves requiring a small input signal, for these will require a smaller value of $R_0'$ for a given $\Delta$ than other valves requiring bigger input signals. Pentodes or tetrodes, then, are more suitable to use in this particular circuit than triodes.

## Input Signal Requirements

From (1) and (7), we have

$$E_1 = E - \frac{g_mR_0'E}{1 + 2g_mR_0'}$$

$$= E\left(\frac{1 + g_mR_0'}{1 + 2g_mR_0'}\right)$$

$$\therefore \quad \frac{E}{E_1} = \frac{1 + 2g_mR_0'}{1 + g_mR_0'}$$

Now the fraction $\frac{E}{E_1}$ gives the value of the total input signal across AB (Fig. 1) over the input signal required by $V_1$. Let $g_m = 10$ mA/volt and $R_0' = 1{,}000$ ohms as before. We have

$$\frac{E}{E_1} = \frac{1 + 2 \times \frac{10}{1{,}000} \times 1{,}000}{1 + \frac{10}{1{,}000} \times 1{,}000} = \frac{21}{11} \approx 1.91$$

so that the input signal required by the circuit is, in these circumstances, 1.91 times that required by one of the push-pull valves. Since $i_1$ is greater than $i_2$, it follows that $V_1$ has a larger input signal than $V_2$ and so overloads first. Suppose $V_1$ can accept a maximum input signal of 8 volts peak value. The input necessary to the circuit is $8 \times 1.91 = 15.28$ volts for maximum output power. The dependence of $\frac{E}{E_1}$ on the value of $g_mR_0'$ is illustrated in Fig. 3, from which it can be seen that the greater $g_mR_0'$ is the more nearly will $E$ approach twice $E_1$.

## Dependence of Output Power on Value of $g_mR_0'$

It is also interesting to find the effect of varying $R_0'$. on the power output of the two valves. Since $i_1$ exceeds $i_2$, it follows that if $V_1$ is driven to the limit of its power output, then $V_2$ is being under-run to an extent governed by the value of $R_0'$. The power actually delivered by the two valves

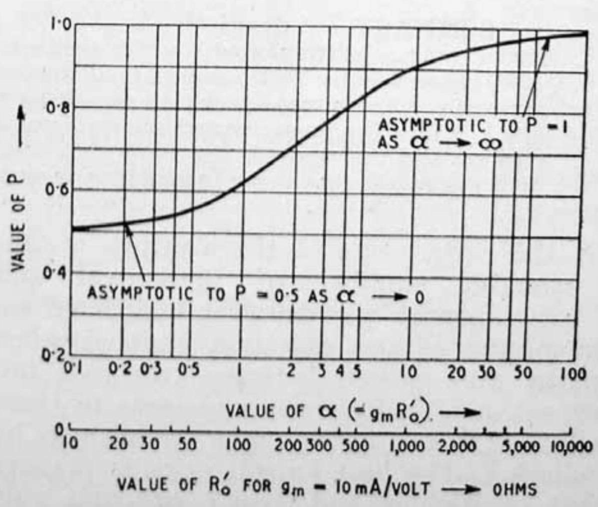


Fig. 4. *Illustrating the dependence of output power ratio on the value of* $\alpha$.

is proportional to $i_1^2 + i_2^2$ whereas if both anode currents equalled $i_1$ the power would be proportional to $2\,i_1^2$. Hence the fraction of the maximum possible power output actually obtained, $P$, is defined by

$$P = \frac{i_1^2 + i_2^2}{2i_1^2}$$

$$= \tfrac{1}{2}\left(1 + \frac{i_2^2}{i_1^2}\right)$$

From (5) and (6)

$$\frac{i_2}{i_1} = \frac{g_m R_0'}{1 + g_m R_0'},$$

$$\therefore \quad P = \tfrac{1}{2}\left[1 + \frac{g_m^2 R_0'^2}{(1 + g_m R_0')^2}\right]$$

$$= \tfrac{1}{2}\left[1 + \frac{\alpha^2}{(1 + \alpha)^2}\right]$$

where $\alpha = g_m R_0'$

In Fig. 4 this fraction has been plotted in the form of a curve and from this we can estimate the value of $\alpha$ necessary to give the desired power output. Suppose, for example, we want at least 90 per cent. of the maximum possible power output. For this value $\alpha = 10$ from Fig. 4, so that, if $g_m = 10$ mA/volt for the particular valves used $R_0' = \frac{\alpha}{g_m} = \frac{10}{\frac{10}{1{,}000}} = 1{,}000$ ohms. Clearly the greater $g_m$ is the less will $R_0'$ need to be to give the required output power.

This method of achieving push-pull operation is covered by British Patents Nos. 492,407 (General Electric Co., Ltd.) and 508,697 (British Broadcasting Corporation).

## REFERENCES


[1] O. H. Schmitt, *Journ. Scient. Instr.*, March, 1938, Vol. 15, p. 100.
[2] W. T. Cocking, *Wireless World*, 13th April, 1939, Vol. 44, p. 340.